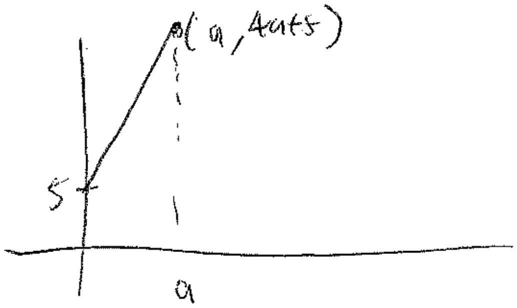
NOTE: 27

should be $h'(t) = 8h(t)$

Week 2
MATH 34B

TA: Jerry Luo
jerryluo8@math.ucsb.edu
Office Hours: Wednesdays 1:30-2:30PM, South Hall 6431X

8. Sketch a diagram showing the area the following integral represents on scratch paper, then use the formula for the area of a trapezoid to calculate the value.
$\int_0^a (4x+5)dx$



$$A_{trap} = (\text{base}) * \left(\frac{\text{height}_1 + \text{height}_2}{2}\right)$$
$$= a * \left(\frac{5 + 4a+5}{2}\right)$$
$$= a\left(\frac{4a+10}{2}\right)$$
$$= a(2a+5)$$

14. What is the area under the graph of the function $f(t) = t^9 + t$ between $t = 0$ and $t = 1$?

$$\int_0^1 t^9 + t \, dt$$
$$= \frac{t^{10}}{10} + \frac{t^2}{2}\Big|_0^1 = \frac{1^{10}}{10} + \frac{1^2}{2} - \frac{0^{10}}{10} - \frac{0^2}{2}$$
$$= \frac{1}{10} + \frac{1}{2}$$

16. Integrate:

(a) $\int_0^1 (2x^4 + 3x^3 + 3x^2 + 2x + 3)dx$

(b) $\int_1^2 (x+3)^2 dx$

(c) $\int_0^1 (ax^2 + b)dx$

a) $\int_0^1 2x^4 + 3x^3 + 3x^2 + 2x + 3 \, dx$

$$= \frac{2x^4}{5} + \frac{3x^4}{4} + \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + 3x \Big|_0^1$$

$$= \frac{2(1)^4}{5} + \frac{3(1)^4}{4} + (1)^3 + (1)^2 + 3(1)$$

$$-(0+0+0+0+0)$$

$$= \boxed{\frac{2}{5} + \frac{3}{4} + 1 + 1 + 3}$$

b) $\int_1^2 (x+3)^2 \, dx$

$$= \int_1^2 x^2 + 6x + 9 \, dx$$

$$= \frac{x^3}{3} + \frac{6x^2}{2} + 9x \Big|_1^2 = \frac{x^3}{3} + 3x^2 + 9x \Big|_1^2$$

$$= \frac{2^3}{3} + 3(2)^2 + 9(2) - \left(\frac{1^3}{3} + 3(1)^2 + 9(1)\right)$$

$$= \frac{2^3}{3} + 3(2)^2 + 9(2) - \left(\frac{1}{3} + 3 + 9\right)$$

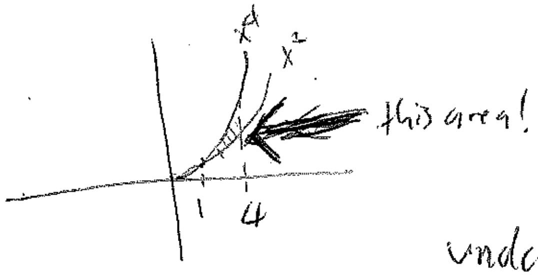
c) $\int_0^1 ax^2 + b \, dx$

$$\frac{ax^3}{3} + bx \Big|_0^1$$

$$= \frac{a(1)^3}{3} + b(1) - (0+0)$$

$$= \frac{a}{3} + b$$

19. Consider the functions $f(x) = x^2$ and $g(x) = x^4$. Find the area of the region between f(x) and g(x) bounded on the left by the vertical line x=1 and on the right by x=4. (Hint: draw a diagram and subtract one area from another.)



This is the area under ~~of~~ x^4 minus area under x^2.

$$= \int_1^4 x^4\,dx - \int_1^4 x^2\,dx$$

$$= \frac{x^5}{5}\Big|_1^4 - \frac{x^3}{3}\Big|_1^4$$

$$= \frac{4^5}{5} - \frac{1}{5} - \left(\frac{4^3}{3} - \frac{1}{3}\right)$$

27. Find a non-zero exponential function h(t) so that $h'(t) = 8h(t)$. (Hint: Look back at the section on differentiating exponential functions.)

Let $y = h(t)$

So, $y' = 8y$.

$$\frac{y'}{y} = 8$$

$$\ln(y) = 8t + C.$$

$$y = e^{8t+C} = e^C e^{8t}$$

$$= Ce^{8t} \text{ (since } C \text{ is constant)}$$

E.g. $C = 1$, $y = e^{8t}$.

OR

Recall: $\frac{d}{dt} e^{kt} = ke^{t}$

In particular,

$$\frac{d}{dt} e^{8t} = 8e^{8t}.$$

So, take $y = e^{8t}$.

29. The temperature T of a cup of coffee is a function T(t) where t is the time in minutes. The room temperature is 15° Celsius. The rate at which the coffee cools down is proportional to the difference between the temperature of the coffee and the room temperature. Use this information to write a differential equation describing the derivative of the coffee temperature in terms of T and t. Use C as your proportionality constant. C should be a positive number. Write T instead of T(t).

$T'(t) \sim (T-15)$ [since room temp. is 15°]

So, $T' = C(T-15)$, for constant C

39. The number of megawatts supplied by a power station at time t is $p(t) = 120 + t^2$ where t is measured in hours. During a 24 hour time interval $0 \leq t \leq 24$ what was the average wattage supplied?

$$\text{Average Wattage} = \frac{1}{t_{final} - t_{initial}} \int_{t_{initial}}^{t_{final}} p(t)\, dt.$$

$$= \frac{1}{24-0} \int_0^{24} 120 + t^2\, dt$$

$$= \frac{1}{24} \left(120t + \frac{t^3}{3}\right)\Big|_0^{24}$$

$$= \frac{1}{24}\left(120(24) + \frac{(24)^3}{3}\right)$$